Wind Instrument Transfer Responses

James W. Beauchamp
(School of Music, University of Illinois at
Urbana-Champaign, Urbana, IL 61801)*

Mouthpiece and output pressure signals have been recorded, harmonic spectra have been computed, and transfer responses have been estimated by taking ratios of corresponding harmonic amplitudes. Each response appears to be a series of points on a smooth high pass filter function, even though it actually corresponds to points on (tending toward the minima of) a multi-resonant response curve. According to simplified assumptions the measured responses would be independent of amplitude and fundamental frequency. However, there is in fact a considerable amount of variation. Results will be presented and possible reasons for the variations will be discussed.

Presented at the 115th Meeting of the Acoustical Society of America, Seattle, Washington, May 20, 1988 (BBB7).

* Analysis portion of this work completed while at the Center for Computer Research in Music and Acoustics, Stanford University.

0. Introduction.

Subtractive synthesis has long been used as a model for the electronic production of musical sounds. This model was based on a long-held belief that it was the basis for behavior of acoustic musical instruments. For example, the idea that the spectral envelope of an instrument's sound is characterized by one or more formants which remain consistent over the pitch range of the instrument is based on the assumption of the validity of subtractive synthesis. In such a model there is an input signal, called the excitation, a filter or transfer function, and an output signal. Given the set of pressure amplitudes $P_{in}(k\,f_1)$ of the harmonics of frequency f_1, the amplitudes of the output signal $P_{out}(k\,f_1)$ can be computed using the equation

$$P_{out}(k\,f_1) = T(k\,f_1)\,P_{in}(k\,f_1) \qquad [1]$$

where T(f) is the transfer response evaluated at frequency f. For a wind instrument P_{in} is measured in or near the mouthpiece, and P_{out} is measured at some distance and orientation outside of the instrument, preferably at some location which minimizes directional effects. It is assumed that T(f) describes a linear system and therefore is independent of frequency, input amplitude, and waveform. Further assumptions are that the filter is time-invariant and can be modeled as a two port device, so that it is not necessary to take into account the impedance of the excitation source. The general block diagram for the subtractive synthesis system is shown in Figure 1. It seemed to be a worthwhile goal to determine realistic transfer response functions for wind musical instruments which could be used to predict the output spectra from input spectra.

Several methods of measuring T(f) are possible. With artificial electroacoustic input one can choose between swept frequency sine wave, impulse, and pseudo-random noise. Each of these techniques can yield the transfer response for arbitrary (continuous) values of f. With human performer input, measurements are restricted to a set of harmonic frequencies, except for the possibility of utilizing small variations around these frequencies or of utilizing the denser spectra associated with attack transients.

Artificial input methods of measurement allow rigorous control of the measurement environment and near-absolute repeatability of results, assuming one can afford the cost of setting up this environment. On the other hand, a good reason for testing with human input is that, even though the results are not strictly repeatable, the circumstances of performance, which are, after all, what we ultimately want to analyze, are more perfectly simulated. Hopefully, theory and practice would agree, and measurements using artificial input could be used to predict the performance outcome. The goals of this paper are to present data comparing information measured for both the artificial and human input cases and to discuss the variability of the human input data in light of what we know about pipe acoustics.

That T(f) basically describes a high pass filter response with superimposed resonances is perhaps not generally appreciated, even though results have appeared several times in the literature. Certainly low pass and band pass filters have been more commonly used in subtractive synthesis applications. Also, while there have been a number of reports showing "transformation functions" and "radiation efficiency" as monotonically increasing graphs and a few reports showing the complete curves with multiple resonances, there have been no instances, to this author's knowledge, which compare or attempt to show the agreement (or lack of agreement) between the two cases, one of the primary themes of this paper.

A parameter of wind instruments which has been more frequently reported in the literature is the input impedance $Z_{in}(f)$, which is the ratio between the input pressure and particle velocity in the steady state. For the case where the wind instrument is lossless we can derive a relation between the magnitude of T(f) and the real part of $Z_{in}(f)$ by equating expressions for input and output power. Input power W_{in} can be stated in terms of $Z_{in}(f)$ and P_{in} as

$$W_{in}(f) = |P_{in}(f)|^2 \, \mathrm{Re}\{1/Z_{in}(f)\} \qquad [2]$$

Assuming that P_{out} is measured in the far field of an anechoic environment where propagation is spherical, we can write the output power W_{out} in terms of P_{out} as

$$W_{out}(f) = 4 \pi r^2 \, |P_{out}(f)|^2/(D(f)\, Z_o) \qquad [3]$$

where r is the distance to the origin of the spherical front, Z_o is the characteristic impedance of air (density times velocity of sound **c**), and D is a directivity index which depends on frequency, the effective radius of radiation **a**, and angular orientation. For low frequencies, below a cutoff frequency of roughly $\mathbf{c}/(\pi\,\mathbf{a})$, we assume that D = 1.
Defining the power efficiency as

$$E(f) = W_{out}(f)/W_{in}(f) \qquad [4]$$

and setting it equal to unity, we can combine Equations 2 and 3 and solve for the ratio of P_{out} to P_{in} to obtain

$$|T(f)| = |P_{out}/P_{in}| = \mathrm{sqrt}[\, Z_o \, D(f) \, \mathrm{Re}\{1/Z_{in}(f)\}/(4 \pi r^2)\,] \qquad [5]$$

Further, if we take D = 1 and assume that $Z_{in}(f)$ is strongly characterized by its extrema where it is very close to being real, for frequencies below cutoff we can approximate

$$|T(f)| = \mathrm{sqrt}[\, Zo /(4 \pi r^2 |Z_{in}(f)|)\,] \qquad [6]$$

I.e., the transfer response should be proportional to the square root of the inverted input impedance function. An examination of swept frequency sine wave measurements of $Z_{in}(f)$ and T(f) for wind instruments indicate that this relationship is definitely not accurate -- the shapes of the two curves are quite different, as can be seen from Figure 2 which simultaneously plots T(f) and $Z_{in}(f)$ for a cornet. (This plot was made for the author by Arthur Benade in his lab at Case Western Reserve University in March, 1973.) Therefore, we must conclude that E is less than unity, and that losses occur within the air column. However, a qualitative inverse relationship between $Z_{in}(f)$ and T(f) still prevails: When $Z_{in}(f)$ reaches a peak, T(f) is at a minimum, and vice versa.

1. Measurements of T(f) previously reported.

The earliest sine wave measurements that I have found are ones made by Daniel Martin as part of his physics doctoral thesis [Martin, 1941]. Figure 3 reproduces Martin's figure which shows "relative response" with and without a mouthpiece for a cornet. While this graph agrees qualitatively with more recent measurements, it does not correspond to our definition of T(f) because it is based on the difference between the sound pressure level on axis and the level at the output of an electroacoustic driver with the connection to the horn pipe removed, rather than the actual mouthpiece pressure level.

I presented measurements of trombone tone mouthpiece and on-axis pressure spectra and the resulting T(f) at the ASA meeting of April, 1969 [Beauchamp, 1969], and these results were later published in conjunction with papers on nonlinear synthesis [Beauchamp, 1979, 1980, 1982.] The results will not be reproduced here in deference to more complete recent analyses of the same data.

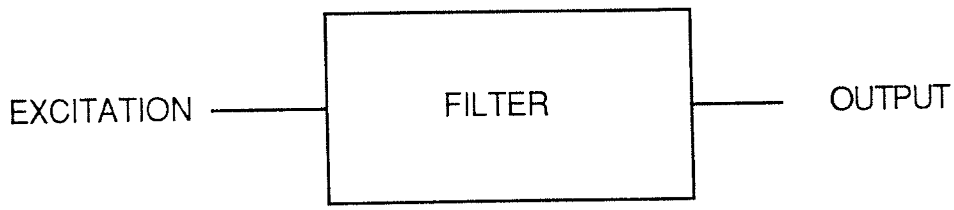

Figure 1. Subtractive Synthesis Model.

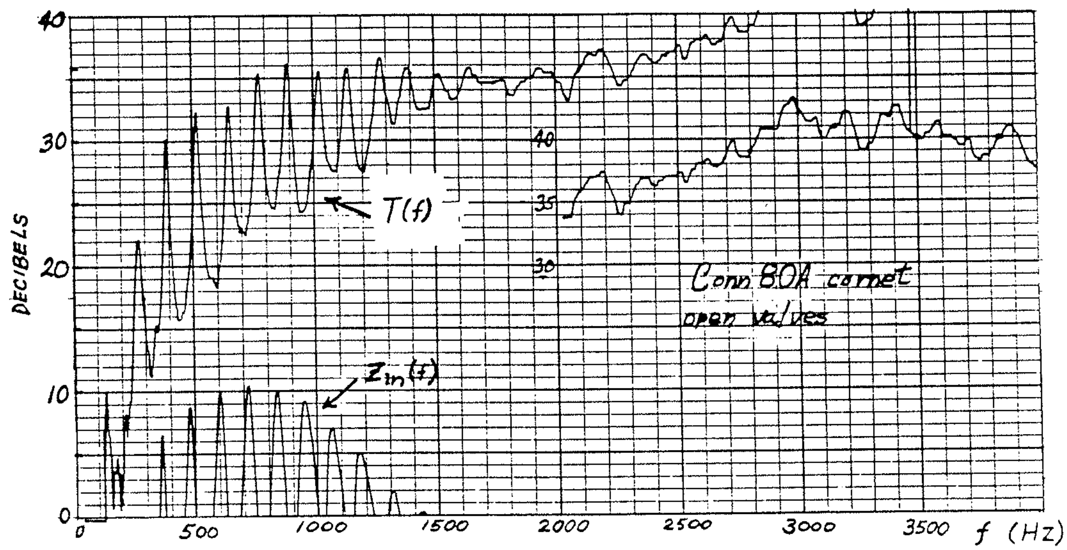

Figure 2. Comparison of Transfer Response T(f) and Input Impedance Response Z_{in} (in dB) for Cornet (Conn 80A, open values).

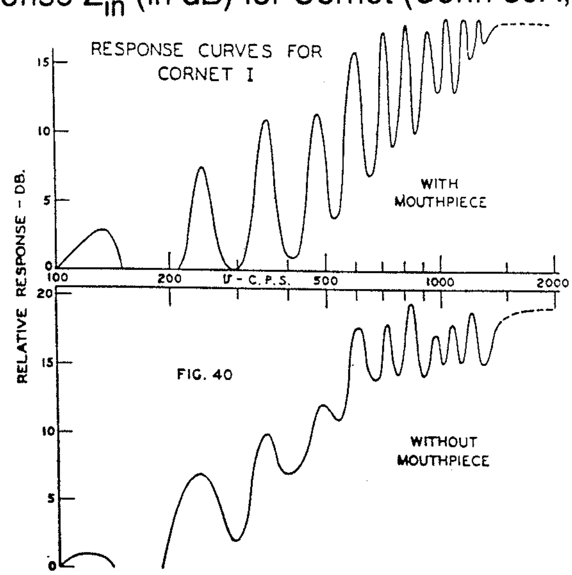

Figure 3. Response Functions for Cornet from Doctoral Thesis of D. W. Martin, p. 69 (1941)

Several transfer functions based on measurements with human input have been depicted by Arthur Benade in an article [Benade, 1973], in a book [Benade, 1976], and, with more detail, in presentations at ASA meetings [Benade, 1980; Keefe and Benade, 1980; Benade, 1981].

Computer-assisted swept sine wave measurements of $Z_{in}(f)$ and T(f) for a trumpet and a trombone were described by Elliott, Bowsher, and Watkinson [Elliott et al, 1982]. They computed what may be the first wind instrument loss vs. frequency curve (E(f), as defined above), clearly showing that power loss decreases with increasing frequency. In decibels, it increases from negative values towards zero at roughly a 6 dB/octave rate, but with considerable (≈±6 dB) variations around that trend.

2. Measurement Technique

Three sets of data for instruments driven by human performers have been collected, one for a trombone (Bach Stradivarius Model 36 with Bach 6.5 AL mouthpiece), one for a clarinet (Buffet with Selmer mouthpiece), and one for an alto saxophone (Selmer Mark 6 with Goldentone 3 mouthpiece and Rico Royal No. 3 reed). In each case tones at several pitches were each played at *pp*, *mf*, *ff*, and (*pp* < *ff* > *pp*) dynamic levels, and the resulting mouthpiece and output pressures were recorded on the left and right channels, respectively, of reel-to-reel half-track recording tape. The conditions for the trombone recordings were much more carefully controlled than for the two woodwinds, so the trombone will be the focus of this preliminary version of the paper. Figure 4 shows a block diagram of the human input measurement system.

The trombone in closed position was mounted on a rigid stand placed in an anechoic chamber. The microphone consisted of a B&K 1613 SLM with omnidirectional 1 inch diameter capsule positioned on axis to the trombone at 6.5 feet (2.0 m.) from the bell. A B&K 4136 1/4 inch condensor microphone was threaded into a collar surrounding the mouthpiece backbore just beyond its cup so that the microphone diaphragm was approximately flush with the inside of the backbore. This was a reasonably secure position for recording under performance conditions, as we had to be concerned about breath and saliva possibly damaging the diaphragm. The 4136 is rated to be linear over a very wide range of pressures (up to 180 dB SPL) and exhibits a flat pressure response over a wide range of frequencies (up to 20 KHz). Microphone outputs were fed into the left and right channels of an Ampex 354 recorder installed outside the anechoic chamber. Four trombonists each played 20 tones consisting of the four dynamics on each of the concert pitches Bb2, F3, Bb3, D4, and F4, and the first trombonist repeated his tone set. Only the *pp*, *mf*, and *ff* dynamics were used for this study. 333 Hz sine tones corresponding to 173 dB (for the mouthpiece) and 110 dB (for the output) were recorded for later comparison purposes.

Each stereo channel was digitized using 16 bits at a 20 KHz sample rate and read into a computer file. Individual tones were extracted from the continuous recording and analyzed using a short-time Fourier transform (STFT) program. The program accomplishes the following steps:

1) Read successive blocks of samples, each of which approximately corresponds to twice the assumed fundamental period N of the tone.
2) Separate the samples into separate blocks for the two channels.
3) Multiply each sample block by a Hanning window function $(1 - \cos(4\pi n/N))$.
4) Interpolate each block to the next lower power-of-2 number of points. (Since the signals were oversampled, linear interpolation was used.)
5) Compute FFT's on each block.
6) Select the even components as the desired harmonics (since analysis occurs over a double period), compute their magnitude values, and convert them to decibels.
7) Average the decibel magnitudes for each channel over the number of blocks read.
8) Compute the transfer response by subtracting the mouthpiece decibel magnitude (channel 1) from that of the output (channel 2) for each harmonic.

For this experiment the number of blocks read corresponded to a segment of 0.1 seconds duration beginning 1 second after the start of the tone. Also, the fundamental phase difference between blocks was computed, and the average frequency error was estimated. This value was used to correct the initial period assumption, and the analyses were repeated with this correction.

A stereo synthetic tone consisting of amplitude 1000 at 116 Hz on one channel and 2000 at 232 Hz on the other channel was used to test the analysis program. When the analysis frequency was set to 116 Hz, the analysis of the principle components appeared to be perfect to within ±0.1 dB, and unwanted components were at least 60 dB down. When the analysis frequency was offset from this amount, an error in components adjacent to the principle ones began to appear. This error was at least 45 dB down for a 1 Hz offset and 38 dB down for a 2 Hz offset. Nonadjacent component amplitudes were at least 20 dB further reduced. There is obviously a smearing effect when the analysis frequency is chosen to be different than the frequency of the tone analyzed, due to the nonideal response characteristic of the Hanning window. This will have some effect on amplitudes of harmonics with much stronger neighbors and increasingly on the upper partials, since the error depends on the product of the harmonic number and the analysis mistuning error. The effect could be lessened considerably by use of a more elaborate window function. However, this would probably have only a small effect on the overall results.

Besides the trombone tones themselves, two other tape segments were analyzed in the same manner for reference purposes. 333 Hz sine tones were recorded on tape along with the trombone tones analyzed in order to establish SPL calibration levels and also to check for tape distortion. The second and third harmonic components were at least 42 dB down from the fundamental, and harmonics above the third were at least 68 dB down, indicating a smooth distortion of about 1% for these tones, which were recorded at levels approximating those of the highest amplitude trombone tones. In addition, a portion of "silence" taken from the interval between two tones was analyzed for several harmonics at each of the five frequencies of the trombone tones. This established a noise floor for each analysis condition. Data points which were not at least 6 dB above the corresponding noise floor values were not considered valid. We observed that the low frequency noise floor was dominated by print-through from other tape layers and the high frequencies were dominated by tape hiss. Most data points were above the noise floor, the exceptions being the fundamental components for the Bb2 tone played *pp* and *mf*, the F3 tone played pp, and some of the very highest harmonics of tones played *pp* and *mf*.

A similar physical setup was used for the swept frequency sine wave measurement. Again, a trombone in closed position was mounted on a stand in an anechoic chamber. The mouthpiece was driven by a University Sound ID-75 electroacoustic driver (rated at 75 watts, 16 Ω) attached to the mouthpiece via a custom coupler. The mouthpiece pressure was sampled by a B&K 4136 1/4 inch microphone, and the output pressure was read by a B&K 4134 1/2 inch microphone. Unfortunately, we could not use the same trombone for this measurement; this one was a Holton tenor. However, the same Bach 6.5 AL mouthpiece was used. The driver was operated at levels between 0 and 30 dBm (0 dBm = .77 volts) which at a frequency of 800 Hz corresponded to SPLs between 142 to 172 dB in the mouthpiece and 78 to 108 dB at the output. As far as decibel measurements were concerned, it was observed that the entire system was linear over this range. A block diagram for this measurement system is shown in Figure 5.

A Wavetek 134 sweep generator was used to drive the electroacoustic driver in linear-f-vs.-time mode. The sweep time was set slow enough (on the order of 1 minute) in order to avoid distortion due to sluggishness of the entire system. Because of the high pass nature of the trombone, which would tend to amplify the distortion components in the sweep generator waveform, the output pressure signal was passed through a voltage-controlled low pass filter set to track the sweep generator and strip off harmonics other than the fundamental. Then the mouthpiece and output signals (suitably amplified) were applied to a custom log ratio detector. We found that our detector tracked equal levels within 0.3 dB and unequal levels within 0.8 dB over a 60 dB range on both inputs. The output of the log ratio detector and the sweep control voltage from the sweep generator were applied to a Hewlett Packard 7035B XY recorder, the most sluggish element in the system, and the transfer response was recorded. This curve was later transcribed to a dB-vs.-log frequency plot (Figure 6).

In both the human and electroacoustic input measurements a value of 0 dB as given on the graphs corresponds to approximately a -60 dB SPL difference between the output and mouthpiece pressure levels.

Data for a clarinet and an alto saxophone were also collected, using convenient but decidedly not as well-defined conditions as for the trombone measurements. In this case a Barcus Berry 1374 piezo-electric microphone was used to read mouthpiece pressures. This microphone was quite inexpensive, very rugged, and could (and still can) easily withstand

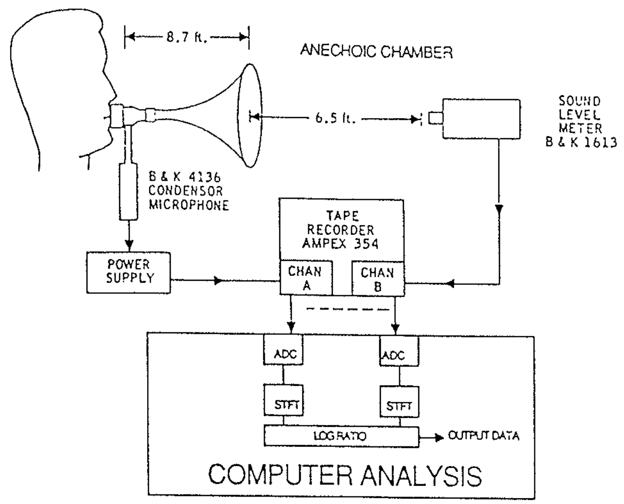

Figure 4. Performer Input T(f) Measurement System.

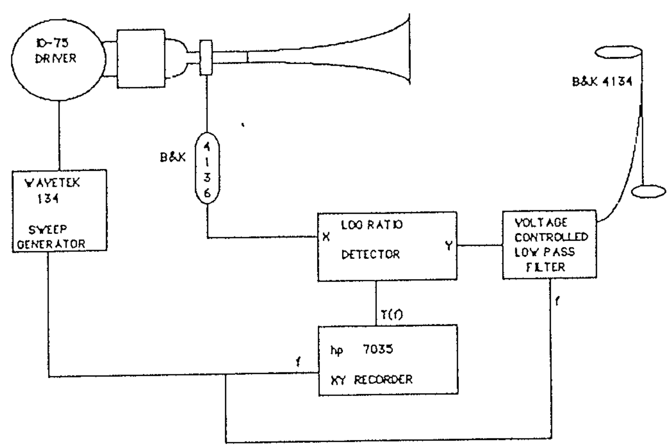

Figure 5. System for Swept Frequency Sine Wave Measurment of T(f)

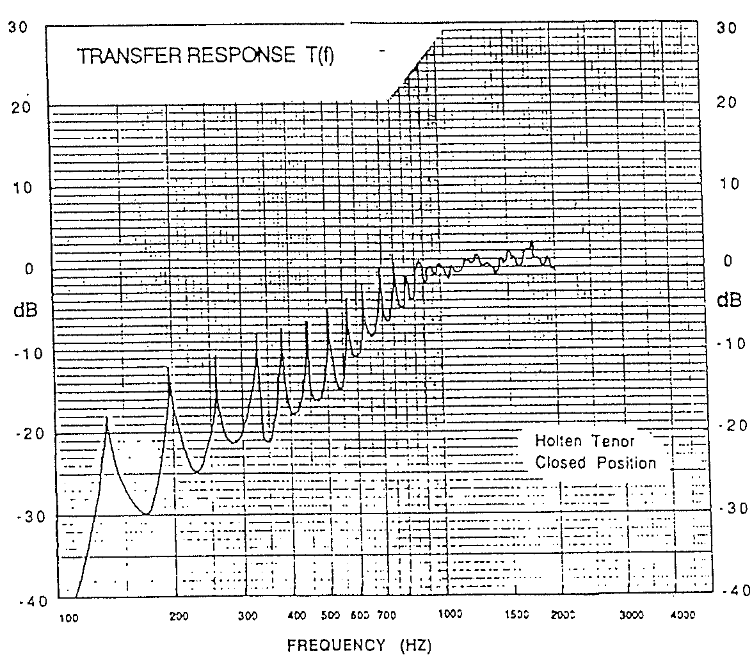

Figure 6. Continuous Frequency Transfer Response of Holton Trombone.

the very high SPL's encountered in wind instrument mouthpieces. However, it came from the manufacturer with no calibration and no specifications were available. A measurement of its frequency response (by Martin Reiling at Northern Illinois University) showed it to be fairly flat between 60 to 3500 Hz (within ±3 dB), but it exhibited wild variations beyond this point. Output pressures were sampled by a Knowles BT-1759 minature electret microphone mounted at the center of a spider which attaches to the bell of the instruments, resulting in the microphone being positioned about 0.3 meters from the plane of the bell. Preliminary analysis results have been obtained, but these will have to wait for a subsequent report.

3. Measurement Results

The result of a swept frequency sine wave measurement of T(f) for the Holton trombone translated from the original dB-vs.-linear-f graph to the a dB-vs.-log-f plot is shown in Figure 6. In the original plot the results of 10 and 20 dBm inputs were overlayed, and it was noted that there were very small differences between the two graphs. It is apparent that below the cutoff frequency (approximately 800 Hz) the transfer function consists of alterations between minima and maxima which are bounded below by an envelope starting at -38 dB at 100 Hz rising to -5 dB at 800 Hz and above by an envelope starting at -21 dB and ending at +1 dB over the same frequency range. This description is imperfect. For example, there is a depression of the actual extrema from these envelopes by amounts varying between 1 to 4 dB in the frequency range 350 to 670 Hz. Also, the lowest frequency minimum occurs well below the lower envelope (about -44 dB at 100 Hz), although it intersects this envelope at about the right point (-35.5 dB at 116 Hz). We also note that the maxima are very cusp-like, while the minima are fairly broad. Then, rather abruptly at 800 Hz the peak-valley swings of the curve lessen and start behaving according to a more complex pattern. Overall, the curve seems to flatten off.

The data for the human performers could be presented many different ways. We chose to present the data on three separate scatter diagrams, as shown in Figures 7 through 9. Each graph corresponds to a different performed dynamic (*pp*, *mf*, or *ff*) and combines measured T(f) points obtained from two performers performing the five pitches (Bb2, F3, Bb3,

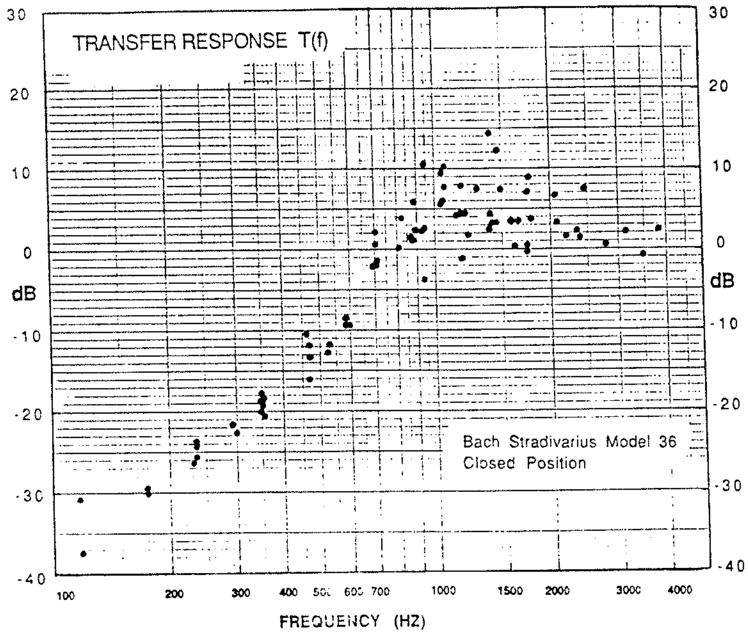

Figure 7.
Transfer Function Data for 10 Analyzed Trombone Tones. Dynamic ***pp***. Pitches Bb2, F3, Bb3, D4, F4; Two Performers.

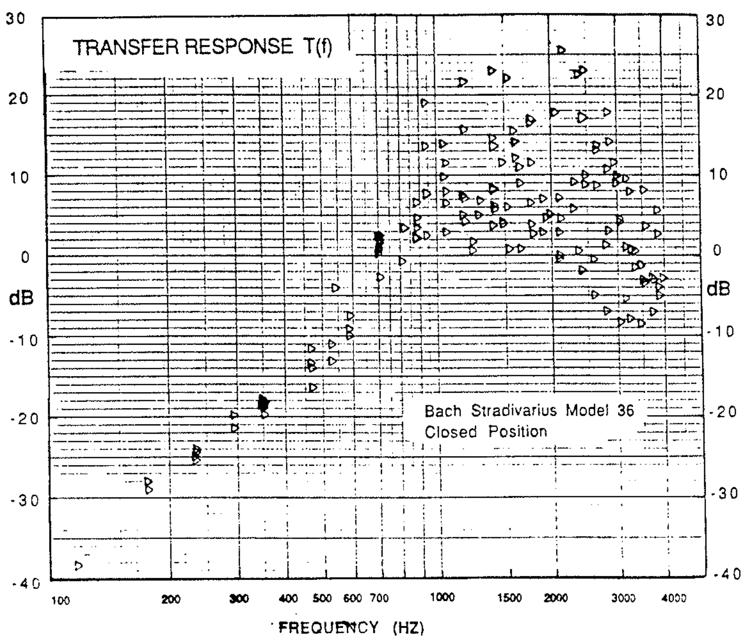

Figure 8.
Transfer Function Data for 10 Analyzed Trombone Tones. Dynamic ***mf***. Pitches Bb2, F3, Bb3, D4, F4; Two Performers.

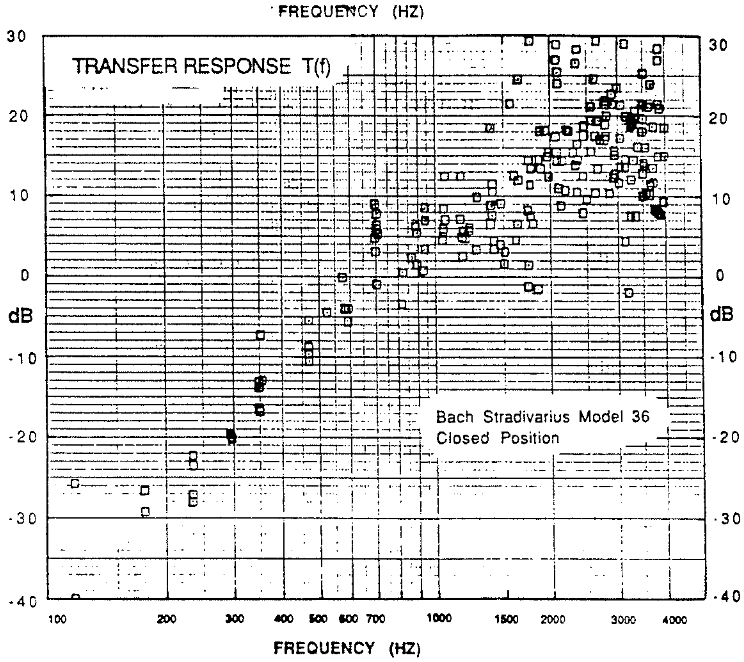

Figure 9.
Transfer Function Data for 10 Analyzed Trombone Tones. Dynamic ***ff***. Pitches Bb2, F3, Bb3, D4, F4; Two Performers.

D4, F4) on that dynamic.

By comparing the plotted performer data with that obtained from the swept frequency technique (e.g., by using transparency overlays), we can see that the agreement below cutoff (f < 800 Hz) is quite good. The fair amount of vertical spread in the performer data below cutoff can be rationalized as resulting from the sampling of the continuous transfer function at slightly different frequencies in the neighborhood of the minima. However, the pitch changes associated with the discrete points do not seem to be enough to account for this difference, so this theory needs to be checked further. Starting at 350 Hz there is a tendency for the *ff* values to be higher than the corresponding *pp* or *mf* values. Then, above cutoff (f > 800 Hz) the agreement between the two methods becomes quite poor. Whereas the continuous response curve flattens off with a 2 or 3 dB ripple around the zero position, the performer data sets are clustered about levels considerably above zero: about 5 dB for the *pp* case, 10 dB for the *mf* case, and 15 dB for the *ff* case. Moreover, the *ff* points seem to be continuing upward at a rate of about 8 dB/octave above 800 Hz, while, mysteriously, the average of the *mf* levels starts to drop above 2000 Hz. Not only is the agreement between the swept frequency and performer data poor above cutoff, but, also, the performer data exhibits a remarkable lack of agreement between the *pp*, *mf*, and *ff* cases.

Under the assumption that the data presented in Figures 6 - 9 was correctly obtained, there seems to be some sort of nonlinear phenomenon which **a**) causes differences between artificially measured T(f)'s and ones measured under performance conditions and, **b**) causes differences between T(f)'s obtained from various performances, which, nevertheless, exhibit appreciable correlation with respect to the dynamic of the performance. However, it is not clear what causes this effect, if it is real. One possible cause is the nonlinear relationship between pressure and volume of air which comes into play at extremely high pressures. Pressure levels in the trombone mouthpiece are as high as 168 dB under performance conditions, as is illustrated by Figure 10. The possibility for harmonic generation in a trumpet air column caused by high pressures was investigated by Backus and Hundley [1971], and they concluded that harmonic generation of this type is very limited. The maximum mouthpiece SPL for their measurements was 155 dB. Indeed, in our own sine wave measurements there appeared to be very little difference between the responses measured with 153 dB SPL input and 163 dB input. Also, as Backus and Hundley mention [1971], while mouthpiece SPL may be very high, the SPL of a standing wave is likely to drop off considerably as it progresses down the length of the pipe. This, however, is only true for frequencies below cutoff. Another possible cause discussed by Elliott, Bowsher, and Watkinson [1982] is the relationship of resistance to flow when particle velocity exceeds a certain limit, where flow suddenly becomes turbulent. Their measurements for a trombone indicate that for loud playing the maximum flow resistance should be about 0.3 MΩ (where 1Ω = 1 Pa-s/m^3). While, as they mention, this is small compared to a typical resonance peak of 20 MΩ, it is at least more appreciable when compared to the impedance above cutoff, which according to their measurements appears to be approximately constant at 10 MΩ.

In conclusion, there appear to be two possible causes for the apparent discrepancies between the measurements of T(f): high pressure for frequencies above cutoff and high turbulent flow. Whether these discrepancies are in fact real and can be attributed to these causes can only be answered with certainty after more research is completed to check the results of this study. The tentative conclusion, based on these results, is that the strengths of the upper partials in trombone tones (at the output) are generally greater above cutoff than would be expected from the linear filter model and that this effect increases with increasing intensity.

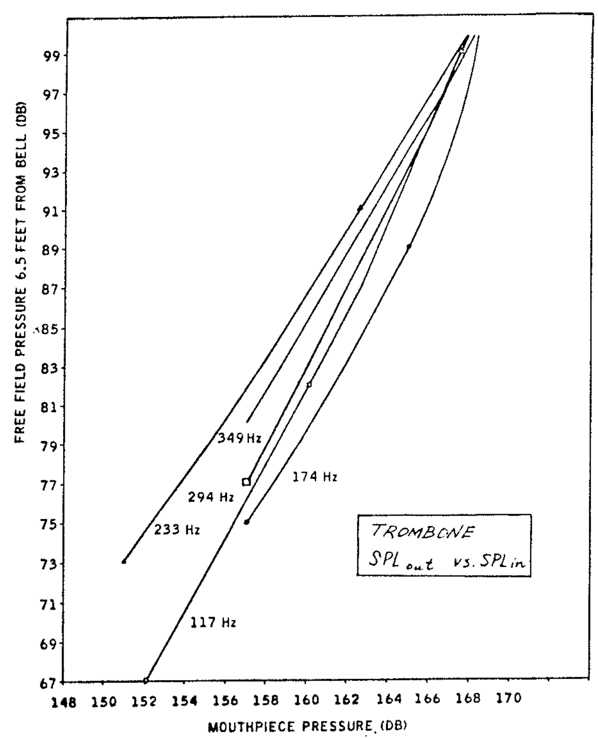

Figure 10. Output Pressure vs. Mouthpiece Pressure for Five Different Pitches Played on a Bach Trombone.

4. References

1941, Daniel W. Martin, "A Physical Investigation of the Performance of Brass Musical Wind Instruments", Ph. D. Dissertation, Univ. of Illinois, p. 69

1969, James W. Beauchamp, "Nonlinear Characteristics of Brass Tones", J. Acoust. Soc. Am., Vol. 46, p. 98 (A).

1971, John Backus and T.C. Hundley, "Harmonic Generation in the Trumpet", J.A.S.A., Vol. 49, pp. 509-519.

1973, Arthur H. Benade, "The Physics of Brasses", Sci. Am., 229, pp. 24-35.

1976, A. H. Benade, Fundamentals of Musical Acoustics, p. 421, Fig. 20.14, p. 481, Fig. 22.5.

1979, J. W. Beauchamp, "Brass Tone Synthesis by Spectrum Evolution Matching with Nonlinear Functions", Computer Music J., Vol. 3, No. 2, pp. 35-43. Reprinted in Foundations of Computer Music, C. Roads & J. Strawn, eds., MIT Press, Cambridge, MA, pp. 95-113.

1980, A. H. Benade, "Musical Spectrum and Transfer Function Measurements: Techniques, Pitfalls, Interpretation", J.A.S.A., Vol. 67, Suppl. 1, p. S97 (A).

1980, D. H. Keefe and A. H. Benade, "Spectrum Transformation Properties of the Clarinet and Saxophone", J.A.S.A., Vol. 67, Suppl. 1, p. S98 (A).

1980, J. W. Beauchamp, "Analysis of Simultaneous Mouthpiece and Output Waveforms of Wind Instruments", Audio Engr. Soc. Preprint No. 1626.

1981, A. H. Benade, J. C. Carman, and P. H. Barrett, "Bassoon Spectrum Transformation Function", J.A.S.A., Vol. 69, Suppl. 1, p. S37 (A).

1982, Stephen Elliott, John Bowsher, and Peter Watkinson, "Input and Transfer Response of Brass Wind Instruments", J.A.S.A., Vol. 72, pp. 1747-1760.