SET THEORY CONCEPTS

The Abstract and the Particular

Octave Equivalence/Enharmonic Equivalence

A category of abstraction whereby pitches that are registal duplicates of one another are considered "equivocable" – ie, belonging to a larger class that includes both of them because they are of the same essence. A440 and A880 are both "A."

A category of abstraction whereby pitches that are the same frequency (or one could say, played with the same key on the piano) are considered "equivocable." B-double-flat (440) and A 440 are the same tone.

The purpose of equivalence (and of abstraction in general) is to reduce out redundant information – to simplify the analytical process and to reveal hidden relations/designs.

Pitch

A particular instance of a pitch as opposed to the abstract essence of the pitch (the thing which several different particular instances share) A^2 or A 440 or G^x 3 instead of simply "A."

Pitch-class

The abstract, universal, or essential *class* of a pitch. The thing about that pitch that different particular instances of it share in common.

Pitch-Intervals

The distance in semi-tones between two particular pitches expressed as an integer (0-11). An *ordered* pitch-interval includes a -/+ which indicates the direction (similar to the concept of vector in mathematics and physics in that it is an expression of both magnitude and direction). In this case, a negative sign indicated a descent in pitch where a positive sign indicates an ascent. An unordered pitch-interval is simply the absolute value |i| of the distance between two particular pitches.

Interval Class

An abstraction of the concept of unordered pitch-intervals which is expressed as the distance between two pitch-classes. Put in another way, because compound intervals are (due to octave equivalence) considered equal to their simple counterparts, and because intervals larger than 6 semitones are considered equivalent to their mod 12 complements, there are only six possible intervallic types or *classes*. For example, one could say that because D2 to B2 (which is a major 6^{th}) and B2 to D3 (which is a minor third) are composed from the same pitches, they have a base sonic similarity or materiality. That base similarity is not destroyed at the most abstract level when you transpose one of those intervals to another minor third, so they belong to a *class*.

Interval-Class Content

The abstract-level total potential of possible intervallic relationships between the members of a pc-set as opposed to the instanced selection of a particular ordering of the pc-set in some musical figure that displays some of the possible interval relations while suppressing others

Interval Vector

A string of six numbers that communicates the interval-class content of a given pitch-class set (or set class). The first number represents the total number of appearances of interval-class 1 that are possible, the second number represents the total number of possible instances of interval-class 2 that are possible, etc. The final number represents interval-class 6 (tritone). Some special properties include pitch-class sets that contain equal numbers of all possible interval classes, sets whose vectors contain unique entries for each interval-class (like for instance, set-class 7-35 (013568T) – the major/natural minor scale).

7-35 (013568T) **interval vector: 254361**

Pitch-Class Sets

An unordered collection of pitch-classes that forms a basis for some kind of compositional activity (used to generate harmonies or melodies). No matter what order/form they take in the music, they will retain their pitch-class and interval-class identity. Composers can also apply some transformational operation to them so as to can create meaningful and coherent musical structures without recourse to a hierarchic tonal system. Notated in [brackets] each member separated with a comma

[2, 4, 6, 2, 8]
[D, E, F#, D, Ab]

Cardinal Number

The number of notes contained in the set. The first number in the designation of a prime form within the Forte-list of set-classes.

Ordinal Number

The second number of the designmation of a prime form within the Forte-List of set classes, representing the set-classes place in the list.

Permutation

a particular ordering of a set of pitches – the incarnation/instantiation of its abstraction. There are n! possible distinct orderings of any pitch-class set.

Circular Permutation

Rotating the order of a pitch-class set so that the second ordering (permutation) of the set begins with the second member of the first permutation of the set after which the order continues as usual.

ABCDE
BCDEA
CDEAB
DEABC
EABCD
ABCDE

Normal Form

A "distilled" permutation of a particular pc-set that is ordered so that the intervals are packed as tightly as possible to the left (beginning) of the set. The distance between the first two integers is the smallest, the distance between the 3rd and first is second smallest, the 4th and first is third smallest, etc. with the smallest intervals on the left of the set (first) and the largest on the right (last). Also, the normal form should be the permutation with the smallest intervallic distance between the last and first interval. It is a compressed version of the pc-set that highlights some important qualities and makes it easy to compare it with other pc-sets in their normal forms.

How to find normal form

1) Excluding redundant pitches, write the pitch-classes in ascending order within an octave. There are as many ways to do this as their are pitches within the set, because this can be done beginning with an of the notes.

2) Choose the ordering that has the smallest interval between the lowest (first) and highest (last) notes.

3) If there is a tie, then choose the ordering with the smallest interval space between the second-to-last and the first pitch, etc.

4) If there is still a tie (for instance in a set like the augmented or diminished triad, where all of the intervals are equidistant) then choose the ordering which begins with the smallest interval.

Mapping

An expression of an operation whereby one set is transformed into another. Transposition, inversion, multiplication, union, etc are examples of mapping.

Transposition

You can transpose a pitch-collection (pc-set) by some number n by adding n to every member of the collection (class). Similarly, you can determine if two pc-sets are related by transposition if you subtract the normal form of one from the other (keeping in mind that for instances where the remainder is a negative number, it should be translated into its complement mod 12 for the sake of comparison). If the remainder of each subtraction is the same number, then the two sets are related by transposition, and the reminder is the "transposition number." The relationship between the two is expressed in the format "set B is T_n of set A" where n is the transposition number. This is a statement known as a "mapping."

ex.

[2, 4, 5, 7]	**set A**
- [5, 7, 8, T]	**set B**
-3 -3 -3 -3	
(9 9 9 9) -- (complement mod 12 of -3)	

So we say that set B is T_9 of set A

Inversion

The complement mod 12 of any pitch is a nominal example of inversion (inversion around the integer 0, or inversion that does not also include transposition). Unlike inversion in tonal music, inversion in atonal music often also involves trnasposition, and so it is expressed by the formula T_nI, (T stands for transposition by some interval n, and I stands for Inversion, or complement mod 12). Traditionally we invert first, then transpose. If you invert a pitch collection twice by the same level n, you will reproduce the original pitch-collection. In order to transpose a pc-set back to its original form, you would have to transpose it by the complement mod 12 of the value for n that you originally used to transpose it.

You can tell if two sets are related by inversion if the intervals between the members of the first set are the same as those in the second set, but in reverse order.

Another way to find out if two sets are inversionally related, and by which value of n, is to compare the sets by determining if there is a consistent **index number**. This is done by reversing the order of the second set, and adding the corresponding elements. If you arrive at the same number for each, then the two sets are related by inversion and the sum (the index number) is equal to n of T_nI.

[7, 8, 11]	[7	8	11]
[1,4,5]	+[5	4	1]
	0	0	0

So we say that set B is T_0I of set A

Prime Form

The "most normal" of normal forms that begins on 0, and is most packed to the left. Notated with parenthesis, without commas: (014).

How to find prime form:

1) Put the set into normal form
2) Transpose the set so that it begins on 0.
3) Invert the original set and repeat steps ½
4) Compare the two transposed normal forms – the one that is most tightly packed to the left is the prime form.

Set-Class

The abstraction of all particular instances of a class of pc-sets that are related to one another either by transposition or inversion. This class of sets is represented by the "prime form". This is an incredibly useful concept that forms the basis of much of set theory on the deeper levels of structure. It is akin to saying "major chord" as opposed to "D major triad." But because there are a great many more possible kinds of harmonic classes in atonal music, the concept of set class has much more to tell us about the unique structures that govern a piece, and the unity that exists behind their various particular instances in a work.

Transpositional Symmetry

This is a property of a set which is able to transpose onto itself at some value of n (other than the nominal instance whereby all sets map onto themselves at T_0). If the interval vector of the set contains an entry equal to the number of notes in the set, or half that number in the case of the tritone, then the set will be transpositionally symmetrical. Compare this to the concept of modes of limited transposition found in Messiaen's theory.

Inversional Symmetry

This is a property of a set which is able to map onto itself through inversion at some value of n. If the intervals between consecutive pitches of the normal form are palindromic (that is, the same from L-R and R-L : 4114, 54345, 717) then the set is inversionally symmetrical.

Invariance

Common Tones are examples of invariance, but the concept is larger. Subsets can be held invariant between two members of the same set-class or two different forms of the same row. Often this is done for the sake of retaining the same materials, just in a different order [to create permutation – to an incredibly sensitive ear, this can be like a sonic pun of sorts]).

Z-Relation

To sets are "Z-related" if they have the same interval vector, but do not belong to the same set class ie, they are not related by either transposition or inversion). The Z-correspondence is a kind of similarity that two sets can share, and therefore a reservoir of common sonic possibilities

Complement Relation

The relationship of literal complementarity exists between two sets where the second set contains all of the pitches of the aggregate that are excluded from the first set. This relationship can be literal or abstract. An example of abstract complementarity would be where the second set is a member of the set-class of the collection of pitches that would be literally complementary to the first set.

ex: Opening figure from *Régard de l'Étoile* by Messiaen; Schoenberg's String Quartet #3.

Superset

A superset is a larger set out of which you can derive smaller sets called subsets whose members are entirely contained in the superset. [1,2,3] is a literal subset of [1,2,3,4,5,6]. There is also such a thing as an abstract subset of a superset where a set which is a member of the same set-class as the literal superset is related to the superset. [7, 8, 9] is an abstract subset of the superset [1, 2, 3, 4, 5, 6]. The union of 2 sets will produce a superset of which both of the original sets will be subsets. {1, 2, 3] + [4, 5, 6] = [1, 2, 3, 4, 5, 6].

Transpositional Combination

Describes the operation by which a superset x is created by transposing a set A by some interval n and combining the transposed and original forms of the set so that they are subsets of superset x (you also reduce out redundant pitches).

Contour Relations

A means of comparing two lines of music by their contours (ie, by the relative highness/lowness of pitches as they appear in the order presented by the line). Contour relations are expressed in strings of numbers called "contour-segments" or "c-seg's." Notated <2013>, indicates that the first pitch is the second highest note (or third-lowest), the second pitch is the lowest pitch, the third pitch is the second lowest (or the third highest), and the final pitch is the highest. There will be as many numbers as there are pitches in the segment of a line that you are analyzing).

Segmentation

Segmentation is the process of making decisions about how to break up the music into sets for meaningful analysis. This can be done in a number of categorical ways (melodic/motivic, harmonic, proximity and similarity, contrapuntal, canonic, etc). This is an important aspect of set analysis and takes time/experience to do instinctively/well.

Derivation

The process by which one pc-set is used to generated others via transposition, inversion, multiplication, complementation, union, or intersection

Scores to examine:

Webern *Fünf Satze für Streichquarttet, op. 5*
Concerto for Nine Instruments, op. 24

Schoenberg *Piano piece op. 11 a*
String Quartet no. 3

Messiaen *Régard de l'Étoile* from *Vingts Regards sur l'"Enfant Jésus*